

MATH REVIEW FOR SONOGRAPHY STUDENTS

STUDENT REFERENCE | Exponents and scientific notation

This review brings together the math used in ultrasound physics: powers of ten, logarithms, algebra, ratios, reciprocals, metric units, decibels, and trigonometric values. Keep units with every calculation and check whether the answer is reasonable.

EXPONENTS AND POWERS OF TEN

An exponent tells how many times a base is multiplied by itself when the exponent is a positive whole number. For example, $10^3 = 10 \times 10 \times 10 = 1,000$. A zero exponent gives 1; a negative exponent means the reciprocal of the corresponding positive power.

Power of ten	Numerical value
10^{-5}	0.00001
10^{-4}	0.0001
10^{-3}	0.001
10^{-2}	0.01
10^{-1}	0.1
10^0	1
10^1	10
10^2	100
10^3	1,000
10^4	10,000
10^5	100,000

Scientific notation: Write a number as a value from 1 up to, but not including, 10 multiplied by a power of ten. Example: $0.00001\text{ V} = 1 \times 10^{-5}\text{ V} = 0.01\text{ mV}$.

OPERATIONS WITH POWERS OF TEN

Operation	Rule	Example
Multiply	$10^x \times 10^y = 10^{x+y}$	$10^2 \times 10^3 = 10^5$
Divide	$10^x \div 10^y = 10^{x-y}$	$10^6 \div 10^4 = 10^2$
Raise to a power	$(10^x)^y = 10^{xy}$	$(10^5)^3 = 10^{15}$
Take the reciprocal	$10^{-x} = 1 / 10^x$	$10^{-3} = 1 / 1,000$

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STUDENT REFERENCE | Logarithms and algebraic properties

LOGARITHMS

A logarithm answers the question, “What exponent produces this number?” For base ten, $\log_{10}(1,000) = 3$ because $10^3 = 1,000$. In this review, log means base ten. The logarithm of a positive number below 1 is negative; $\log_{10}(1) = 0$. Logarithms of zero or negative numbers are not defined in real-number arithmetic.

Power	Number	$\log_{10}(\text{number})$
10^4	10,000	4
10^3	1,000	3
10^2	100	2
10^1	10	1
10^0	1	0
10^{-1}	0.1	-1
10^{-2}	0.01	-2
10^{-3}	0.001	-3
10^{-4}	0.0001	-4

ALGEBRAIC PROPERTIES

Maintain equality by applying the same valid operation to both sides. Division by zero is never allowed.

Property	Rule
Addition	If $a = b$ and $c = d$, then $a + c = b + d$.
Subtraction	If $a = b$ and $c = d$, then $a - c = b - d$.
Multiplication	If $a = b$, then $ca = cb$.
Division	If $a = b$ and $c \neq 0$, then $a / c = b / c$.
Substitution	If $a = b$, either may replace the other in an expression.
Reflexive	$a = a$
Symmetric	If $a = b$, then $b = a$.
Transitive	If $a = b$ and $b = c$, then $a = c$.
Distributive	$a(b + c) = ab + ac$

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STUDENT REFERENCE | Ratios, reciprocals, and proportionality

RATIOS AND PERCENTAGES

A ratio compares two quantities. Write it as $a : b$ or a / b , and state the order of the comparison. Two female patients for every five male patients gives a female-to-male ratio of $2 : 5$. That is different from the female share of all seven patients, which is $2 / 7$.

To convert a ratio to a percentage, divide the first value by the reference value and multiply by 100. For a test score, $75 / 75 = 1 = 100\%$ (unity). A score of $50 / 75 = 0.666... \approx \mathbf{66.7\%}$. Keep extra digits until the final answer, then round as requested.

RECIPROCAL

The reciprocal of a nonzero number x is $1 / x$. Multiplying a number by its reciprocal gives 1: $x \times (1 / x) = 1$. For example, the reciprocal of 4 is $1 / 4 = 0.25$, and $4 \times 0.25 = 1$. Zero has no reciprocal.

Sonography connection: Frequency and period are reciprocals: $f = 1 / T$ and $T = 1 / f$. Use hertz with seconds, or matching reciprocal prefixes such as megahertz with microseconds. For example, 5 MHz corresponds to a period of 0.2 μs .

DIRECT AND INVERSE RELATIONSHIPS

A **direct relationship** means two quantities change by the same factor when other relevant quantities are held constant. An **inverse relationship** means one quantity changes by the reciprocal factor: doubling one halves the other.

Relationship	Equation and example
Direct: distance and velocity (time held constant)	$D = V \times T$ 60 miles/hour $\times$ 1 hour = 60 miles 70 miles/hour $\times$ 1 hour = 70 miles
Direct: distance and time (velocity held constant)	$D = V \times T$ 60 miles/hour $\times$ 2 hours = 120 miles
Direct: time and distance (velocity held constant)	$T = D / V$ 60 miles $\div$ 60 miles/hour = 1 hour 120 miles $\div$ 60 miles/hour = 2 hours
Inverse: time and velocity (distance held constant)	$T = D / V$ 60 miles $\div$ 60 miles/hour = 1 hour 60 miles $\div$ 120 miles/hour = 0.5 hour

Pulse-echo reminder: $D = V \times T$ describes travel distance. For reflector depth using round-trip echo time, $\text{depth} = c \times \text{time} / 2$.

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STUDENT REFERENCE | Metric values, conversions, and reciprocal pairs

METRIC PREFIXES

A metric prefix multiplies the unit by a power of ten. Symbols are case-sensitive: **M** means mega, while **m** means milli. The base unit has no prefix. The table lists the prefixes included in this review.

Prefix	Symbol	Factor	Numerical value
giga	G	10^9	1,000,000,000
mega	M	10^6	1,000,000
kilo	k	10^3	1,000
hecto	h	10^2	100
deca	da	10^1	10
none	none	10^0	1
deci	d	10^{-1}	0.1
centi	c	10^{-2}	0.01
milli	m	10^{-3}	0.001
micro	μ	10^{-6}	0.000001
nano	n	10^{-9}	0.000000001

CONVERT THE VALUE AND KEEP THE SAME QUANTITY

Multiply by the old prefix factor, then divide by the new prefix factor. A smaller unit produces a larger numerical value; a larger unit produces a smaller numerical value.

Examples: 5 MHz = 5,000,000 Hz; 250 μ s = 0.250 ms; 8 cm = 0.08 m. With powers of ten: $250 \times 10^{-6} \text{ s} \div 10^{-3} \text{ s/ms} = 0.250 \text{ ms}$.

RECIPROCAL PREFIX FACTORS

Factor	Reciprocal factor	Pair
10^9	10^{-9}	giga $\leftrightarrow$ nano
10^6	10^{-6}	mega $\leftrightarrow$ micro
10^3	10^{-3}	kilo $\leftrightarrow$ milli
10^2	10^{-2}	hecto $\leftrightarrow$ centi
10^1	10^{-1}	deca $\leftrightarrow$ deci

These are reciprocal **numerical factors**, not interchangeable units. Reciprocals also invert the unit: 1 / second = hertz. Thus, $1 / (2 \text{ ms}) = 0.5 \text{ kHz} = 500 \text{ Hz}$.

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STUDENT REFERENCE | Decibel notation and attenuation

DECIBELS COMPARE A VALUE WITH A REFERENCE

A decibel (dB), one tenth of a bel, expresses a logarithmic ratio. In sonography it is used for acoustic power or intensity, attenuation, echo amplitudes, receiver dynamic range, and the range of signal levels mapped to displayed gray shades. The reference value must be stated or understood.

Power or intensity: change in dB = $10 \log_{10}(I_2 / I_1)$.

Amplitude or voltage: change in dB = $20 \log_{10}(A_2 / A_1)$.

Use the amplitude relationship for comparable signals under the same impedance conditions. Both values in a ratio must use the same units.

Intensity example: For a chosen reference of 1 mW/cm², an intensity of 100 mW/cm² gives $10 \log_{10}(100 / 1) = 20$ dB. The reference of 1 mW/cm² is for this example; it is not a universal ultrasound reference.

Attenuation example: A decrease from 100 to 10 mW/cm² gives $10 \log_{10}(10 / 100) = -10$ dB. This is a signed change of -10 dB, or an attenuation loss of 10 dB. For a voltage decrease from 100 to 10 μV, the corresponding change is -20 dB.

INTENSITY REMAINING AFTER A DECREASE

Intensity change	Intensity remaining	Intensity lost
0 dB	100%	0%
-1 dB	79.4%	20.6%
-2 dB	63.1%	36.9%
-3 dB	50.1%	49.9%
-6 dB	25.1%	74.9%
-10 dB	10%	90%
-20 dB	1%	99%
-30 dB	0.1%	99.9%
-40 dB	0.01%	99.99%

Values are rounded. A 3 dB decrease is approximately half the intensity; a 6 dB decrease is approximately one quarter. A 10 dB decrease leaves one tenth. These percentages describe **intensity**, not amplitude.

Calculator tip: Calculate the ratio first, take log₁₀ of that ratio, then multiply by 10 or 20. Do not use the natural-log (ln) key for these formulas.

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TRIGONOMETRIC REFERENCE VALUES

Sine and cosine relate angles to side lengths in a right triangle. For an acute angle θ , $\sin \theta = \text{opposite side} / \text{hypotenuse}$ and $\cos \theta = \text{adjacent side} / \text{hypotenuse}$. The values below are rounded to three decimal places.

Angle (degrees)	Sine	Cosine
0°	0.000	1.000
5°	0.087	0.996
10°	0.174	0.985
15°	0.259	0.966
20°	0.342	0.940
25°	0.423	0.906
30°	0.500	0.866
35°	0.574	0.819
40°	0.643	0.766
45°	0.707	0.707
50°	0.766	0.643
55°	0.819	0.574
60°	0.866	0.500
65°	0.906	0.423
70°	0.940	0.342
75°	0.966	0.259
80°	0.985	0.174
85°	0.996	0.087
90°	1.000	0.000

Sonography connection: The Doppler relationship includes $\cos \theta$, where θ is the angle between the sound beam and flow direction. At 0°, cosine is 1; at 60°, it is 0.5; at 90°, it is 0. Use **degree mode** on your calculator when working with this table.

Before submitting a calculation: Check the units, decimal placement, prefix, and rounding. State which quantities are held constant when describing a proportional relationship.

Prepared from Jim Baun, Math Review - Addendum (2025), and Metric Values and Reciprocals. Prefix symbols and decibel conventions checked against NIST SP 811, Chapters 4 and 8: [nist.gov/pml/special-publication-811](https://www.nist.gov/pml/special-publication-811).